Estimation of Radon Priority Areas – sources of error and uncertainty

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Content

• Motivation:
  Why is this so important?

• Sources of uncertainty

• Consequences & how to deal with?
Indoor radon - essentials

odourless, tasteless, colourless...

let’s skip this part...
Reminder: RPA definitions

Some examples of operable RPA definitions, based on different Rn measures:

- An area $B$ (grid cell, municipality...), in which the mean population-weighted indoor concentration $C$ exceeds the reference level (RL); $AM_B(C) > RL$; measure $= AM_B$
- same, but indoor concentration in *dwellings on ground floor*
- An area $B$, in which the probability that $C$ exceeds the RL, is greater than $p$ (typically 10%); $prob_B(C > RL) > p$; measure $= prob_B$
- The areas $B$ which represent the upper 10% of $AM_B(C)$; measure $= percentile$
- An area, in which the collective exposure (e.g., $AM_B(C) \times$ population) is among the upper 10%

Multinomial:
Instead of 2 classes (RPA / non-RPA), several classes of “Rn-prioritiness”; approach chosen by some countries.

Multivariate:
Although the BSS definition relies on indoor Rn concentration, one may chose to base estimation on other Rn-related variables instead or additionally. Examples: geogenic Rn potential, $U$ concentration in the ground, terrestrial gamma dose rate, geological unit, tectonic features etc.

Important:
There is no “natural” definition of RPA! Therefore, also no “true” RPA! RPAs always depend on definition and to some extent, on estimation method. This is partly a political decision, partly a pragmatic one (i.e., availability of data).
No matter how defined...
RPAs have to be estimated ...
... done with available data ...
... using a certain method.

Data, estimation method and actual performing estimation are prone to errors and uncertainty.
Data

Data are always dirty, noisy, incorrect, erroneous, incomplete, ill-defined, uncertain.

Data as observations:
- measurement uncertainty (not only counting uncertainty! Sampling and measurement procedures include uncertainty, sometimes this is the most important part, but difficult to quantify)
- “semantic” uncertainty
  (Value reported ground-floor measurement, in fact first floor,...)
- wrong (e.g., geology wrongly classified)
- sloppiness errors (manual copying of data, wrong insertion into table, Excel misreads decimal point, x and y coordinates confused,...)

Data as samples from a population:
- not representative
  (relevant if the target is a statement about the population!)
- finite / limited sample size ⇒ estimation uncertainty
Sample size effect: A numerical experiment, 1

The municipality Gigritzpatschen (AT), Rn concentrations in all N=1004 houses. Quite realistic!

In a survey, we cannot measure all houses, but a number n, selected randomly. I.e., a representative sample in the best case.

Declare an area (U) RPA, if in U: \( \text{prob}_{U}(z>200)>0.1 \)

Areas U: quadratic fractions of the municipality.

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**True RPA status**

orange = RPA
blue = non-RPA
Sample size effect:  
A numerical experiment, 2

Finite population! (Statistically → sample without replacement)

**Question:** For given sampling rate, assuming representative (random) sampling, which is the error rate of estimated RPA status?

Method: many virtual “sampling campaigns” (2000-5000 realizations), calculate FP and FN rates of estimated RPA status

Even for high sampling rate, error chance can be high! This is the case, if a cells contains few houses / if true variability is high / if true p is close to class limit.

![Graph showing sampling rate vs. error rates](image-url)
Variability and uncertainty

A quantity $Z$ can be truly variable in space or time... (typical for Rn quantities, as for most environmental quantities!)

Take a sample of the quantity from a given space or time interval, $\{z_1, ..., z_n\}$

True mean of $Z$ ... $M$, true SD .... $S$

sample mean $\text{AM}(z_i)$ .... $m$, sample $\text{SD}(z_i)$ ... $s$, estimates of $M$ and $S$.

Uncertainty of the sample mean .... $\text{unc}(m) = s/\sqrt{n}$

= consequence of variability ($s$) (sample size, QA)

Variability = natural, irreducible!
Uncertainty of estimate: can be reduced (sample size, QA)
Methods – 1: complexity

Methods are almost always simplistic, idealization of a situation, make possibly unrealistic assumptions.

simplistic: not all relevant controls considered

some typical unrealistic / idealistic assumptions:
- normal distribution (also LN often idealization!),
- homoscedasticity,
- statistical independence (to be able to use CLT),
- infinite sample,
- sample with replacement from finite population while it should be without,
- uncertainty of predictor ignored
  (regression! – leads to biased estimates of regression coefficients)
Ex.: simplification & sample size

Real region, from Metro Rn WP4 mapping exercise

histogram – not really LN, but approximately

well known... AM is unbiased (=accurate), but precision depends on sample size

pink: \( p^* := \text{prob}_{B}(Z > z) = t_{n-1}(\zeta + \sqrt{\frac{n}{n+1}}); \quad \zeta := \frac{\ln z - AM_B(\ln Z)}{SD_B(\ln Z)} \)

red: SE\(\sim\)\(\sigma/\sqrt{n}\)
blue: sampling without replacement

red: \( p = \Phi(\zeta) \)
blue: simulation, counting instances
Normally, “the true” method does not exist. In many cases, different methods or models are conceivable, which are all “more or less” correct. Results may be different.

Naturally, one would look for the “best” model, to be chosen by some validation procedure.

But:

- Different validation criteria may lead to preference of different models. (Accuracy, precision, 1\textsuperscript{st} / 2\textsuperscript{nd} kind error rates, cross-validation correlation, RMSE, other metrics?)
- The best available model may not be the best possible (which is not known).
- To avoid overfitting, one may remove predictors
- For practical reasons, one may opt for a compromise between model complexity and correctness.

Choice of model $\rightarrow$ structural uncertainty
### Tentative Taxonomy of estimation approaches

*certainly not complete!*

<table>
<thead>
<tr>
<th>spatial correlation not considered</th>
<th>spatial correlation considered</th>
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<tbody>
<tr>
<td><strong>univariate</strong></td>
<td></td>
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<tr>
<td>• sample stat, stat(z)</td>
<td>• geostat. model &amp; cut-off, I(Z*)</td>
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<td>• enhanced by assuming univar.</td>
<td>• indicator kriging (hard/soft), I*(z)</td>
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<td>distribution, e.g. LN, stat’(z)</td>
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<td>→ cut-off, I(stat(z))</td>
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<tr>
<td><strong>multivariate</strong></td>
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<tr>
<td>• ANOVA type</td>
<td>• co-kriging et al. &amp; cut-off, I(Z*;y)</td>
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<tr>
<td>• logistic-type regression, logi(z)=g(y)</td>
<td>• regression kriging &amp; cut-off, I(f*(y))</td>
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<tr>
<td>• geographically weighted, local</td>
<td>• indicator regression kriging,</td>
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<tr>
<td>regression etc.</td>
<td>I*(f(y))</td>
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<td>→ cut-off, I(f(y))</td>
<td>• indicator co-kriging, I*(z;y)</td>
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<tr>
<td>• full bivariate through copula</td>
<td>• machine learning</td>
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<tr>
<td>• cross-classification</td>
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I – indicator according RPA class definition; more complicated for multinomial class → I₁, I₂,..  
* - interpolation, e.g. kriging type  
z, y – primary and secondary variables
Example: European indoor Rn data, 1

Enhanced empirical exceedance probability in cell B by LN modelling, given n data Z in B:

\[ p := \text{prob}(Z > 300 \text{ Bq/m}^3); \] RPA criterion: \( p > 0.1 \) estimated from cell statistics:

\[ p' := \text{prob}_B(Z > z) = t_{n-1}\left(\frac{z}{\sqrt{\frac{n}{n+1}}}\right); \quad \zeta := \frac{\ln z - \text{AM}_B(\ln Z)}{\text{SD}_B(\ln Z)} \]

**univariate**

\[ p^{**} = I_{0.1}(p'*(Z)) \]

\[ p^{**} = I_{0.5}(I^{*0.1}(p'(Z))) \]

**bivariate, \( Y = \text{U conc.} \)**

\[ p^{**} = I_{0.1}(p^*(f_{\text{logi}}(Y))) \]

\( f_{\text{logi}} \) - logistic regression
Example: European indoor Rn data, 2

bivariate cross-classification $p'$ against $U$ concentration in cells B

ROC curve close to diagonal: association between $p'$ and $U$ not very strong!

blue: $\text{prob}(Z>300)<10\%$ with $1-\alpha$ confidence
orange: $>10\%$ with $1-\beta$ confidence
grey: undecided

... large areas because of weak association class limits with 90\%C.I. (by bootstrap)

lower: $U=1.43\text{ ppm (1.28...1.57)}$
upper: $U=3.27\text{ ppm (3.00...3.73)}$

lower: $U=1.66\text{ ppm (1.50...1.88)}$
upper: $U=2.71\text{ ppm (2.55...2.89)}$
Methods – 3: parameterization

Models must be parameterized:
- Regression models: coefficients estimated from data
- Cross-classification: rule derived from training object

Therefore, parameters are uncertain: ⇒

Predictions are also uncertain.
Estimation uncertainty – real data!

Proposal for RPAs, Germany, based on cross-classification method.

**Primary variable:** Z = indoor Rn concentration in ground floor dwellings, houses with basement;

**Secondary variable:** Y = Geogenic Rn potential (GRP). Modelled by SGS on U = 10 km × 10 km grid, geology as deterministic predictor.

**RPA definition:** grid cell U = RPA, if \( p = \text{prob}_U(Z > 300 \text{ Bq/m}^3) > 3 \times \) German average \( \approx 10\% \).

\( p \) estimated by enhanced empirical exceedance prob., assuming LN within cells, GSD = \( \exp(\text{SD}_U(\ln Z)) = 2 \):

\[
p = t_{n-1} \left[ \frac{n}{\sqrt{n + 1}} \frac{\ln(300) - AM(\ln Z)}{SD(\ln Z)} \right]
\]

(unfortunately biased estimator)

Cell U labelled RPA or non-RPA with confidence 90%, i.e. 1. and 2. kind error probability <0.1.

**RPA:** Y > 44.5 (12.0% of territory);
**Non-RPA:** Y < 20.2 (49.8% of territory);
**Yellow:** undecided
For estimation uncertainty component of model uncertainty:
Ignore unc. of input variables!
Only unc. of association $Z \sim Y$!

By bootstrap ($k=20,000$):
reddish hues: **RPA**: $\text{CI}_{90} = (38.2, 52.8)$
greenish hues: **Non-RPA**: $\text{CI}_{90} = (13.1, 26.4)$

Distribution somewhat unexpected
Probably because classification is ‘very’ nonlinear transform
Summary: Sources of uncertainty

- **Data uncertainty**
  - Intrinsic: data as observations;
  - Data as samples

- **Model uncertainty**
  - Structural uncertainty: choice of model
  - Simplification uncertainty
  - Parameter estimation uncertainty
3 Levels of QA

1. **Design QA:** sampling such that the target (e.g. AM of the population in an area) can be met with given tolerance
   ⇒ sample size, representativeness

2. **Data QA:** correct measurement!!
   Classical metrology QA

3. **Evaluation QA:**
   - select proper method / model
     (easier said than done!)
   - consider, as far as feasible (because this can be complicated), model-induced uncertainty.

Aspects of 2 and 3 discussed in Metro Radon!
Finally: Why is this so important?

• Whether an area is assigned RPA or non-RPA (or a certain level of priorityness) can make a big economic difference:
  - implementation of building norms
  - measurement campaigns
  - remediation
  - property value

• Also possibly legal consequences, if RPA status is legally disputed by stakeholders!

• Administrations and decision makers want to be on the safe side – understandably.
Conclusions & To-do

• RPA definition and estimation: not only academic exercise, but practically important. May have severe economic & political impact. Heavy stakeholder interest! Therefore: QA very important!

• Uncertainty of RPA status (in terms of classification error rate, 1st/2nd kind error prob) has many sources of different types!

• Unc(RPA) can be high, in particular for spatial units close to class limits.

• **To do: Further explore uncertainty budget of RPA!**

• My impression: Unc(RPA) not taken sufficiently seriously!

• **Open questions** which are a big headache in practice:
  - how to communicate the fact that RPAs are “random objects”?
  - how to deal with RPA uncertainty in administrative decision-making?

RPA – a sensitive subject!
Action required in RPA can be costly → political disputes
Thank you!

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